POSTURE MEASUREMENT AND STRUCTURAL PARAMETERS CALIBRATION ON PARALLEL 6 DOF PLATFORM

Wu Jiangning, Zhang Lanfang and Li Shilun
The State Key Lab of Fluid Power Transmission and Control, Zhejiang Univ.
Jnwu@sfp.zju.edu.cn

ABSTRACT

This paper presents a brief introduction about how to calibrate the platform's actual structural parameters by measuring a series of the platform's varying postures, discusses some measures taken in the platform's postures measurement and structural parameters calibration in detail, and gives the experiment results of the platform's posture control before and after the calibration. The results show that the platform's posture control accuracy after the calibration is improved notably.

KEYWORDS

Parallel 6 DOF platform, theodolite, posture measurement, nonlinear equation group, structural parameters calibration

1. INTRODUCTION

Because of its large structural rigidity, strong carrying capacity, high posture accuracy (having no accumulating error of serial manipulator) and quick response speed, the parallel 6 DOF platform is now applied widely not only in the case of innervations simulation such as flight simulator, automobile driving simulator and star field travelling amusement machine and so on, where the requirement of posture accuracy is relatively low, but also in the case of RVD emulator, new type of 6 DOF machine tool [2] and assembling manipulator [3], where the requirement of posture accuracy is relatively high.

In the case where requires relatively high posture accuracy, such as"VARIAX machine tool" made by American GIDDINGS & LEWIS company, whose machining accuracy reaches 5 μm, it is meaningless to improve posture accuracy by improving steady state accuracy of each actuator's extension simply, for manufacture tolerance and assembling tolerance of the platform are main factors of the platform's posture error.

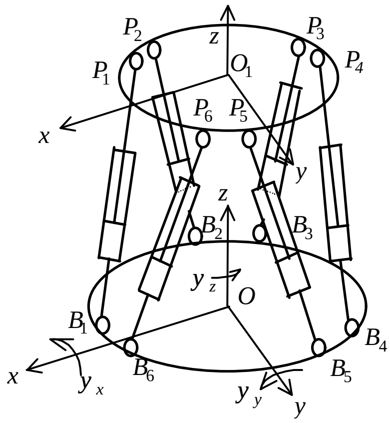

Fig. 1 Configuration of the platform

The configuration of the platform is shown in Fig. 1. The platform's posture motion is carried out by the synergistic movement of all actuators. According to Fig. 1 the relation between each posture vector of the platform can be described in equation (1)

$$\boldsymbol{l_i} = \boldsymbol{T}\boldsymbol{P_{mi}} + \boldsymbol{R} - \boldsymbol{B_i} \quad (i=1,2,\dots6) \qquad \dots(1)$$

where $\boldsymbol{l_i}$ represents the length vector of actuator i (i.e. $\boldsymbol{B_i}\boldsymbol{P_i}$), $\boldsymbol{T}$ the rotary transformation matrix of the platform; $\boldsymbol{P_{mi}}$ (i=1,2,...6) the vector from the origin(O_1) of the moving coordinate system to joint P_i on the platform, $\boldsymbol{R}$ the vector from the origin O of the fixed coordinate system to the origin(O_1) of the moving coordinate system, $\boldsymbol{B_i}$ (i=1,2,...6) the vector from the origin O of the fixed coordinate system to the joint B_i (i=1,2,...6) on the base. $\boldsymbol{R}$ and $\boldsymbol{T}$ contain the 6 degree of freedom posture vectors of the platform $\boldsymbol{X} = (x, y, z, \psi_x, \psi_y, \psi_z)^T$. By using a ψ_z (yaw), ψ_y (pitching), ψ_x (roll) order, the transformation matrix is

$$\boldsymbol{T} = \begin{bmatrix} c\psi_z c\psi_y & c\psi_z s\psi_y s\psi_x - s\psi_z c\psi_x & c\psi_z c\psi_x s\psi_y + s\psi_z s\psi_x \\ s\psi_z c\psi_y & s\psi_z s\psi_x s\psi_y + c\psi_z c\psi_x & s\psi_z c\psi_x s\psi_y - c\psi_z s\psi_x \\ - s\psi_y & c\psi_y s\psi_x & c\psi_y c\psi_x \end{bmatrix} \dots(2)$$

where the abbreviations "s"="sin" and "c"="cos".

From equation (1), we know that the platform's posture is relevant to the position of each joint on the platform and the base, and the length of each actuator. The platform's posture control is realized by calculating the length of each actuator from the given platform's posture instructions based on the inverse solution of equation (1). There are a few of factors that affect the platform's posture control accuracy such as the actuator's extensions, the length between two joints of each actuator when the platform in zero posture, and the coordinates of the joints P_{mi} and B_i (i=1,2,...6) to the moving coordinate system and the fixed coordinate system respectively. Among those the latter three values are the platform's structural parameters. Each actuator's control accuracy decides the actuator extension accuracy, while the accuracy of manufacture and installation affects the structural parameter's accuracy. In the case where high posture control accuracy is required, each actuator's control accuracy can be usually obtained easily. So the main problem is how to minimize the posture error caused by the coordinates tolerance of P_{mi} and B_i (i=1,2,...6) and the length tolerance between two joints of each actuator when the platform is in zero posture. All tolerances are resulted from manufacture and installation.

There are two approaches to solve this problem. One is to improve manufacture and installation accuracy of the platform directly, which will result in the cost of the platform ascending in geometric series when manufacture and installation accuracy is improved every order of magnitude. The other is first to manufacture and install the platform according to the requirement of general accuracy and then to manage to measure the structural parameters, i.e., the actual value of the length between two joints of each actuator when the platform remains in zero posture and the coordinates of the P_{mi} and B_i (i=1,2,…, 6) in moving and fixed coordinate system respectively, and then to utilize software compensation. Its cost is as much as the platform with general accuracy. The article adopts the latter approach that is calibrating the structural parameters and introduces the situation and the result of the measurement based on the theory and emulation of reference [1].

2. THEORETICAL BASE OF PLATFORM'S STRUCTURAL PARAMETER CALIBRATION

Assuming l_i is made up of actuator's nominal length l_{0i} which can be obtained by actuator position sensor and its length deviation Dl_i, from equation (1), we get

$$(l_{0i} + Dl_i)^2 = \left\| \boldsymbol{TP_{mi}} + \boldsymbol{R} - \boldsymbol{B_i} \right\|^2 \qquad i=1,2,\ldots,6 \quad \ldots(3)$$

where Dl_i and $\boldsymbol{B_i}$ are unknown, while l_{0i}, $\boldsymbol{T}$ and $\boldsymbol{R}$ are measurable. Thus, as long as enough groups of l_{0i}, $\boldsymbol{T}$ and $\boldsymbol{R}$ are measured and with the above equation, the values of Dl_i, $\boldsymbol{P_{mi}}$ and $\boldsymbol{B_i}$ can be calculated. The unknown Dl_i is a redundancy, after obtaining $\boldsymbol{P_{mi}}$ and $\boldsymbol{B_i}$, with the above formula Dl_i can be calculated. Because formula (3) is nonlinear, it is not easy to eliminate Dl_i that will complicate the calculation. Hanqi Zhuang and Zvi S. Roth's method [4] shown in Fig. 2 is adopted here that is keeping the i*th* actuator's length fixed when the other actuators extend according to the instructions. In this way, we can calculate $\boldsymbol{P_{mi}}$ and $\boldsymbol{B_i}$ by obtaining enough platform postures. When l_i is fixed, we have

$$\left\| \boldsymbol{T^j P_{mi}} + \boldsymbol{R^j} - \boldsymbol{B_i} \right\| = \left\| \boldsymbol{T^k P_{mi}} + \boldsymbol{R^k} - \boldsymbol{B_i} \right\|$$

$$i = 1,2,\Lambda\ ,6;\ j,k \in \{1,2,\Lambda\ ,m\},\ j < k \qquad \ldots(4)$$

where m represents the number of the platform's postures when i*th* actuator's length is fixed and other's are changed, $\boldsymbol{R^j}$, $\boldsymbol{T^j}$, $\boldsymbol{R^k}$ and $\boldsymbol{T^k}$ are the j*th* and k*th* measured vector from the origin O of the fixed coordinate system to the origin(O_1) of the moving coordinate system and transformation matrix respectively. Assuming s is the number of equations in expression (4), then it follows $s = (m-1)m/2$.

Assuming $f_j = f_j(\boldsymbol{P_{mi}}, \boldsymbol{B_i}) = \left\| \boldsymbol{T^j P_{mi}} + \boldsymbol{R^j} - \boldsymbol{B_i} \right\|^2$, formula (4) can be rewritten as

$$f_j = f_k \qquad j,k \in \{1,2,\Lambda\ ,m\},\ j < k \qquad \ldots(5)$$

Because $\boldsymbol{P_{mi}}$ and $\boldsymbol{B_i}$ contain six unknowns totally and the above expression contains min {m-1, 6} independent equations, it is required that the number of the platform's postures m equals 7. Thus, there are s=21 equations satisfying the condition in expression (5). Hereon we select 6 independent equations as follows to make up of an equation set.

$$f_j = f_{j+1} \qquad j=1,2,\ldots,6 \qquad \ldots(6)$$

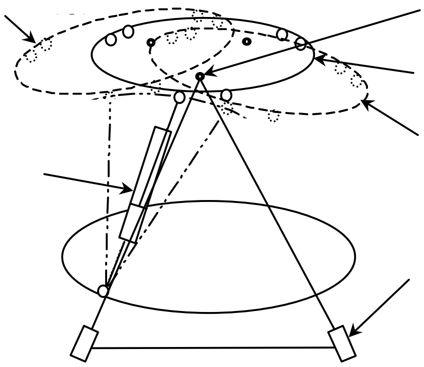

Fig. 2 The survey map of platform's static posture measurement The platform's posture varies when the ith actuator's length is fixed

The problem of how to obtain the i*th* actuator's parameters P_{mi} and B_i in numerical method can be described as how to seek the values of P_{mi} and B_i so that the values J_i of the objective function minimize, where

$$J_i = \sum_{j=1}^{6}\left(f_j - f_{j+1}\right)^2 \qquad i=1,2,\ldots,6 \quad \ldots(7)$$

From formula (3), let l_i be the average of l_i, which yields

$$l_i = \frac{1}{m}\sum_{j=1}^{m} f_j^{1/2} \qquad i=1,2,\ldots,6 \quad \ldots(8)$$

The nonlinear optimization problem of expression (7) can be solved applying Newton method.

According to reference[1], suppose $x_i = \left(P_{mi}^T, B_i^T\right)^T$ and
$h_i(x_i) = \left[h_{i1}(x_i), h_{i2}(x_i), \ldots, h_{i,m-1}(x_i)\right]^T$ where
$h_{ij} = f_j(x_i) - f_{j+1}(x_i)$ i=1,2,...,6; j=1,2,...,m-1
...(9)

In formula (9) h_i is the ith actuator's $(m-1)\times 1$ vector, and x_i is 6×1 vector representing the i*th* actuator's parameter vector. *m*>7 in order to decrease the influence of certain posture random error on measured results and improve the robustness of the actuator's parameter P_{mi} and B_i measured. So the iteration to solve the problem can be expressed as follows:

$$\begin{cases} dx_i^k = -\nabla h_{iL}^{-1}(x_i^k)h_i(x_i^k) \\ x_i^{k+1} = x_i^k + dx_i^k \end{cases} \quad \ldots(10)$$

where ∇h_{iL}^{-1} is the generalized inverse matrix of ∇h_i and is obtained by using the least squares procedure to solve contradictive equation set.

$$\nabla h_{iL}^{-1} = (\nabla h_i^T \nabla h_i)^{-1}\nabla h_i^T \quad \ldots(11)$$

$\nabla h_i = \partial h_i / \partial x_i$ is a $(m-1)\times 6$ Jacobian matrix its jth (j=1,...,m-1) row is

$$\nabla h_{ij} = \left[\left(\partial h_{ij}/\partial P_{mi}\right), \left(\partial h_{ij}/\partial B_i\right)\right] \quad \ldots(12)$$

where

$$\frac{\partial h_{ij}}{\partial P_{mi}} = 2\left((R^j)^T - B_i^T\right)T^j - 2\left((R^{j+1})^T - B_i^T\right)T^{j+!} \quad \ldots(13)$$

$$\frac{\partial h_{ij}}{\partial B_i} = -2P_{mi}^T\left((T^j)^T - (T^{j+1})^T\right) - 2\left((R^j)^T - (R^{j+1})^T\right) \quad \ldots(14)$$

Iterative steps:

1 Selecting the nominal value x_i^0 of x_i as the initial value of the iterative variable;
2 Using the given x_i^k to calculate $h_i(x_i^k)$, $\nabla h_i(x_i^k)$, $\nabla h_{iL}^{-1}(x_i^k)$ and dx_i^k;
3 Calculating x_i^{k+1} and J_i^{k+1};
4 If $J_i^{k+1} < \varepsilon_1$ or $\left|J_i^{k+1} - J_i^k\right| < \varepsilon_2$ end the iteration where ε_1 and ε_2 are positive target values, or $x_i^k = x_i^{k+1}$ and jump to step 2.

3. THE PLATFORM'S POSTURE MEASUREMENT & STRUCTURAL PARAMETERS CALIBRATION

3.1 THE MEASUREMENT OF THE POSTURE

The measurement sketch is shown as Fig. 2. We use two theodolites with 3-D measurement to measure the spatial coordinates of three characteristic points on the platform. It is better to make the three points and xy plane of the moving coordinate system coplanar, otherwise when calculating the platform's posture we have to do coordinate transform. Because of the limit of the site space, we select the DJ6 model theodolites made in Nanjing mapping instrument factory whose angle resolution is 6" and blind range is about 1.4m. It is verified by measuring standard block that with this method the measurement error of the three points' coordinates is less than 0.5mm, which can satisfy the requirement of the present measurement basically. In order to reduce the influence of measurement random error on the result, more than 20 different platform's postures is measured to each actuator when its length is fixed and 20 postures (m=20) whose measurement errors of the three points' distance are lesser are selected to calibrate the parameters.

The main problem of the measurement is how to obtain the platform's posture using the spatial coordinates of the three feature points. The relation of P_i's coordinate $\boldsymbol{P_{mi}}$ in the moving coordinate system and coordinate $\boldsymbol{P_i}$ in the fixed coordinate system ($\boldsymbol{P_i}$ is a point on the platform) is as follows:

$$\boldsymbol{P_i} = \boldsymbol{TP_{mi}} + \boldsymbol{R} \qquad ...(15)$$

The non-linearity of the transformation matrix $\boldsymbol{T}$ brings difficulties to obtain the posture. Here we make use of the special point's method:

- By using the relation of the special point and the origin O_1 of the moving coordinate system, O_1's coordinate vector $\boldsymbol{R}$ in the fixed coordinate system can be obtained;
- By using the particularity of $\boldsymbol{T}$ in formula (2) and the special points on the xy plane in the moving coordinate system, the platform's angular orientation $(\psi_x, \psi_y, \psi_z)^T$ can be calculated.

For example, first select $(1,0,0)^T$ in the moving coordinate system, then multiplies $\boldsymbol{T}$ on left, we have $(c\psi_z c\psi_y, s\psi_z c\psi_y, -s\psi_y)^T$. When the coordinate of this point in the fixed coordinate system and $\boldsymbol{R}$ are given, with the aid of equation (15), ψ_y and ψ_z can be calculated. In a similar way, if select $(0,0,1)^T$ in the moving coordinate system, with equation (15), we can also calculate ψ_x easily.

3.2 STRUCTURAL PARAMETERS CALIBRATION

How to calculate the corresponding actuator's structural parameters through groups of measured platform's postures is the key to the measurement and calibration. During the practical operation, if using the above iteration simply, it is difficult to obtain satisfying result. The method is modified partly according to author's practical experience.

(a) Use descending method to improve step 3 and 4 in the iteration and let descending factor $t \le 1$ to insure $J_i^{k+1} < J_i^k$, otherwise end the iteration.

(b) Newton method requires an exacting initial value, and during iteration we have to take the parameter's nominal value for initial value. But we can change the order of the platform's posture data set to affect the orientation of the iteration. As to 20 groups of postures we can get $20! = 2.432902 \times 10^{18}$ different permutations and combinations. It is obvious that the workload is very heavy if we do iteration operation to every combination. Therefore we utilize computer to array randomly and do nearly 200,000 times of permutation and iteration and select the actuator's structural parameters corresponding to the minimum of the objective function.

(c) Modify the objective function. Even if the structural parameters are fixed, different data permutation of posture data set will have different values of the objective function. Thus we should modify the objective function in expression (7). The modification of the calibration is as follows: Assuming $f_{ave} = \frac{1}{m}\sum_{j=1}^{m} f_j$ then $J = \sum_{j=1}^{m}(f_j - f_{ave})^2$.

(d) Set up the boundary conditions. Make sure the deviation range of the nominal value allowed by the structural parameters to control the iteration and avoid the unbelievable result.

4 COMPARISON OF THE RESULT BEFORE CALIBRATION WITH THAT AFTER CALIBRATION

In order to verify the obtained parameters, we put the result of the calibration to the platform control program and measure the platform's posture again. The results are contrasted with the one whose control program uses the nominal value of the parameter. There are three control instructions as:

(a) Translate in X orientation. X(mm): 600.0 450.0 300.0 150.0 0.0 -150.0 -300.0 -450.0 -600.0 . The other postures remain in zero .The results are shown in Fig. 3(a) and (b).

(b) Rotate around Y-axis. ψ_y (deg) 20.0 15.0 10.0 5.0 0.0 -5.0 -10.0 -15.0 -20.0 . The other postures remains in zero. The results are shown in Fig. 4(a) and (b).

(c) Compound motion. The results of moving from the platform's zero posture to $(100.0,100.0,100.0,5.0,5.0,5.0)^T$ are shown in table 1.

According to the platform's single degree of freedom stroke X ±600mm, Y ±600mm, Z ±250mm, y_x ±23deg, y_y ±23deg, y_z ±60deg , if using the parameters before calibration to control the platform to move along X axis or rotate around Y axis , then the maximum of relative error of the platform's posture is 2.6% in the range of the stroke which is produced on y_x when rotating around Y axis . While using the parameters after calibration to control the platform move along X axis or rotate around Y axis, the maximum of the relative error of the platform's posture in workspace is 0.35% which is produced on X when moving along X axis. The posture error after calibration descends notably.

The data in table 1 also indicates that in compound motion the control results after calibration are better than that before calibration, especially in rotation control.

Table 1

	X (mm)	Y (mm)	Z (mm)	y_x (deg)	y_y (deg)	y_z (deg)
Instruction signal	100.0	100.0	100.0	5.0	5.0	5.0
Results before calibration	99.95	99.66	99.31	4.94	5.22	5.51
results after calibration	100.29	99.79	100.15	5.02	5.06	5.06

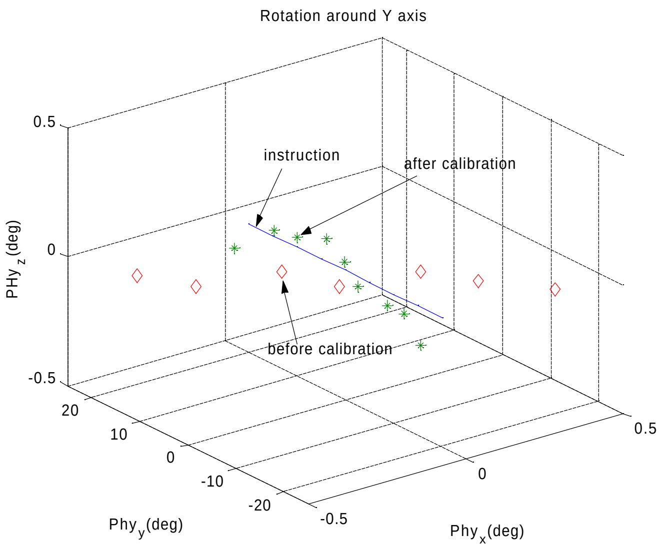

Fig. 3(a)

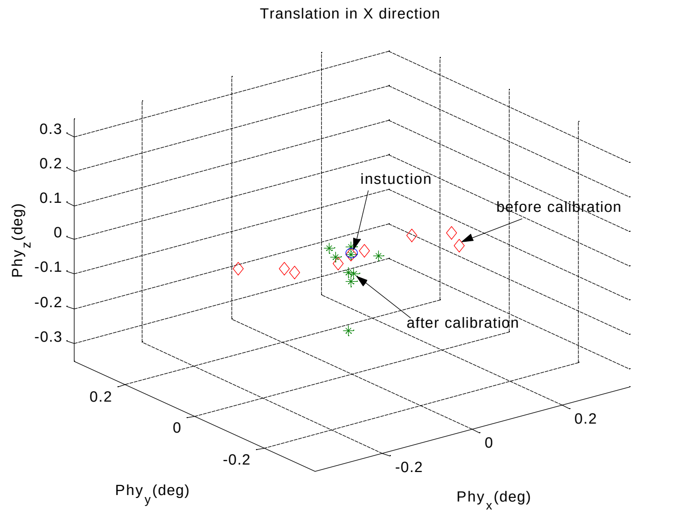

Fig. 3(b)

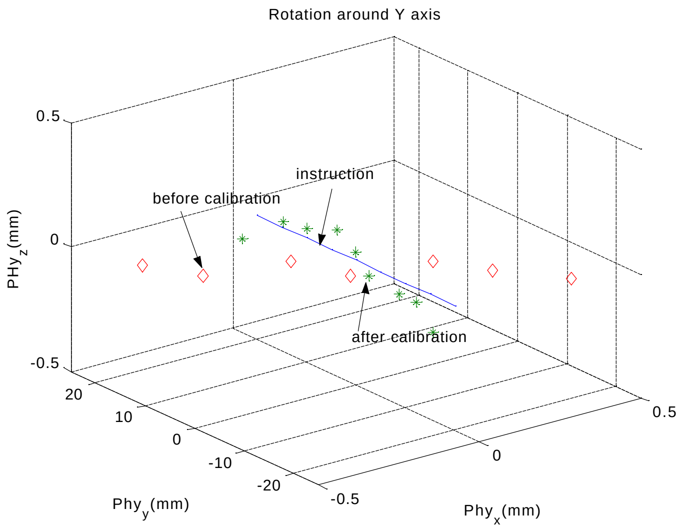

Fig. 4(a)

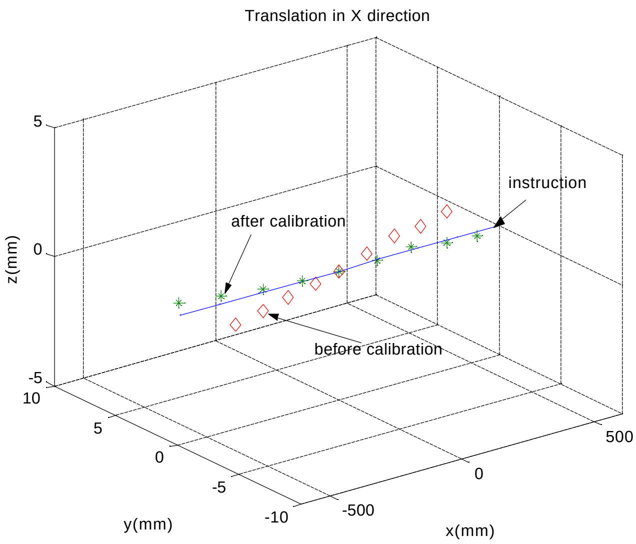

Fig. 4(b)

5 CONCLUSIONS

The experiment measures the platform's postures by using two theodolites 3-D measurement and calibrates the platform's structural parameters. As a result the platform's posture control accuracy is improved .The experiment has attained the expectant aim and verified that it is feasible to improve the platform's posture control accuracy by measuring the platform's postures and calibrating the structural parameters in practice. . The author anticipates if take measures as follows:

(1) Improve the resolution of the theodolite from current 6" to 2" to improve the platform's posture measurement accuracy.

(2) Improve the arithmetic of iteration to solving the nonlinear equation set and increase the number of data groups to each actuator during iteration to get iteration result more close to actual value.

Then the platform's posture control accuracy can be improved further. The method presented here is very worthy for practical application.

REFERENCES

[1] Wu Jiangning, Zuo Aiqiu, Li Shilun. Mechanical Error Analysis and Measurement of Parallel 6 Degree of Platform (in Chinese). China Mechanical Engineering 1999 6 614-617

[2] Peter Baley, Paul Sheldon, Ed Kirkham. A Machine for the 21st Century. Machinery and Production Engineering. 1995(1): 20-27

[3] C.C Nguyen S.S Antrazi Z-L. Zhou and C.E. Campbell, Jr. Experimental Study of Motion Control and Trajectory Planning for a Stewart Platform Robot Menipulator. Proceedings of the 1991 IEEE International Conference on Robotics and Automation. Sacramento, Califomia-April 1991.

[4] Hanqi Zhuang and Zvi S. Roth. Method for Kinematics Calibration of Stewart Platforms. Journal of Robotic System v10 n3 pp.391-405,1993