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CFD METHOD AS AN ANALYSIS TOOL OF FLUID POWER DEVICES

1. Introduction

Even many decades and ages of researches, fundamental problems connected with power devices flow performance are still unsolved. It is very simple to measure pressure (or energy) drop, but it is not so simple to give an answer *why*. In many cases we told about device design, that *it is wrong because it generates big velocity gradients*, but nobody knows *how big is it* and *where is it* – probably only computational fluid dynamics (CFD) gives the answer.

2. Fluid in Flow

A starting point for describing arbitrary flow phenomena is the formulation of a governing equation system, usually the basis of mass and momentum conservation principle. When we consider unsteady and viscous fluid flow it formulates in terms [5, 6]

$$\begin{cases} \dfrac{\partial u_i}{\partial x_j} = 0 \\ \rho \dfrac{\partial u_i}{\partial t} + \rho u_j \dfrac{\partial u_i}{\partial x_j} - \dfrac{\partial \sigma_{ij}}{\partial x_j} = 0 \end{cases}, \tag{1}$$

where the local stress tensor component is

$$\sigma_{ij} = -p\delta_{ij} + 2\eta d_{ij} , \tag{2}$$

and the rate of deformation tensor

$$d_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \tag{3}$$

For the number of (ordinary) fluids, the stress tensor is found varying linearly with shear rate accordingly to Newtonian law, with constant coefficient of viscosity η.

In view of numerical applications it may be appropriate to couple mass and momentum balance equations by penalty function method [1, 4]

$$p = -\lambda \frac{\partial u_i}{\partial x_i}, \quad \lambda \to \infty, \tag{4}$$

in consequence, the phenomenological model forms

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} - \frac{\partial}{\partial x_j}\left(\lambda \frac{\partial u_k}{\partial x_k} \delta_{ij} + \eta \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right) = 0 . \tag{5}$$

3. Model Approximation

The governing equation system (5) makes partial differential equations – in consequence – it can not be solved directly. Equations have to be transformed in a discrete form [3]:

- in time domain, generally by finite difference scheme: one step scheme

$$\frac{\tilde{u}_i^{[n+1]} - u_i^{[n]}}{\Delta t} + \Theta f\left(\tilde{u}_i^{[n+1]}(x_i, t)\right) + (1 - \Theta) f\left(\tilde{u}_i^{[n]}(x_i, t)\right) = 0, \tag{6}$$

  as explicit Euler (Θ=0), implicit Euler (Θ=1), Cranck-Nicholson (Θ=1/2), or more stable and accurate multistep scheme, such as this very effective

$$\frac{\frac{3}{2}\tilde{u}_i^{[n+1]} - 2u_i^{[n]} + \frac{1}{2}u_i^{[n-1]}}{\Delta t} + f\left(\tilde{u}_i^{[n+1]}(x_i, t)\right) = 0, \tag{7}$$

  implicit two step scheme;

- in the spatial discretisation the following techniques are used: finite difference, and similar in some way, finite volume or boundary element, and – especially invited in case of complex geometry – finite element method, when approximation $\tilde{u}_i$ of u_i are specified by linear combination

$$\tilde{u}_i = U_{i\alpha} \Psi_\alpha, \tag{8}$$

over nodes α sub-domains (finite) element $\Omega^{(e)}$

$$\overline{\Omega} = \bigcup_e \Omega^{(e)}. \tag{9}$$

Therefore, we receive numerical approximation of model equations (5) in a weighted residual approach [6]

$$\begin{aligned} &\rho \int_\Omega w_\alpha \Psi_\beta \, d\Omega \, \frac{\frac{3}{2} U_{j\beta}^{[n+1]} - 2U_{j\beta}^{[n]} + \frac{1}{2} U_{j\beta}^{[n-1]}}{\Delta t} + \\ &\rho \int_\Omega \tilde{u}_j \, w_\alpha \frac{\partial \Psi_\beta}{\partial x_j} d\Omega \, U_{i\beta}^{[n+1]} + \lambda \left(\int_\Omega \frac{\partial w_\alpha}{\partial x_i} \frac{\partial \Psi_\beta}{\partial x_j} d\Omega \right) U_{j\beta}^{[n+1]} + . \\ &\eta \int_\Omega \frac{\partial w_\alpha}{\partial x_j} \frac{\partial \Psi_\beta}{\partial x_j} d\Omega \, U_{i\beta}^{[n+1]} = 0 \end{aligned} \tag{10}$$

Weight functions are selected, in basic formulation of weighted residual FEM models, as $w_\alpha = \Psi_\alpha$, but unfortunately in the case of hydraulic devices, the problem of choice of w_α usually more complex. It is well-known that in a convection dominated flow, a solution are often corrupted by a spurious node-to-node oscillations. Therefore, a numerical model requires stabilisation, which can be accomplished by:

- reduced integration procedure, which applies reduced order of quadrature in nodes [2, 4]

$$\xi_i = ctgh\left(Pe_{\xi_i}^{(e)}\right) - \frac{1}{Pe_{\xi_i}^{(e)}}, \tag{11}$$

  which gives generally excellent stability but worse accuracy;
- selection of weighting function $w_\alpha \neq \Psi_\alpha$, typically in form of Petrov-Galerkin (or better Stream Upwind P-G), accordingly to Hughes [2]

$$w_\alpha^{(e)} \equiv \Psi_\alpha^{(e)} + \kappa^{(e)} \frac{\tilde{u}_j^{(e)}}{\tilde{u}_k^{(e)} \tilde{u}_k^{(e)}} \frac{\partial \Psi_\alpha^{+(e)}}{\partial x_j^+}, \tag{12}$$

  where

$$\kappa^{(e)} = \frac{\xi_i^{(e)} \tilde{u}_{\xi_i}^{(e)}}{\sqrt{15}}, \tag{13}$$

  characterised by good stability and efficiency, but slower then first one.

Numerical model equations (10) together with (11) or (12, 13, 11) has to be completed by boundary condition in form of [1, 6]:

- Initial $\tilde{u}_i|_{t=0} = u_{i0}$;
- spatial Dirichlet $\tilde{u}_i|_{wall} = 0$ (on the channel wall) $\tilde{u}_i|_{in} = u_{i\,in}$ (on up-stream boundary), and usually von Neuman stress-free $\tilde{\sigma}_{ij}n_j|_{out} = 0$ (on down-stream boundary).

4. Concluding Remarks

Contrary to other method of investigation, computational fluid dynamics provides detailed information about microscale evolution of flow fields. At the post-processing stage, the data produced by a solver program can be used in the analysis of:

- distribution and evolution of velocity $\vec{u}$ and pressure p fields;
- location and form of hydraulic shadow and recalculation regions;
- other complex parameters calculated at post-processing stage, such as rates of deformations d_{ij} components (33) or spatial gradient of velocity field $\nabla\sqrt{u_i u_i}$

The CFD method makes possibilities of extensive exploration and improvement of fluid power devices. It offers the effortless, but also inexpensive way of estimation and examination of respective flow performance of an arbitrary device.

References

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